Technical Comments

Comments on "Self-Excited Explosive-Driven MHD Generator"

SHIMSHON FRANKENTHAL*
American Science and Engineering Inc., Cambridge, Mass.

IN a recent technical note, Loeffler, Aronowitz, and Ericson[1] formulate the differential equation of the "self-excited" MHD generator (the dot indicates differentiation with respect to time):

$$(C_2 - x)\ddot{x} = 2\dot{x}(\dot{x} - C_1)$$
$$x(0) = 0 \qquad \dot{x}(0) = 1 \qquad \ddot{x}(0) = \Delta$$

and present 1) an exact solution to this equation in the limit of infinite magnetic Reynolds number ($C_1 = 0$), 2) numerical solutions for large magnetic Reynolds numbers and $C_2 = 1.1$, and 3) a qualitative discussion of the effect of the parameters C_1, C_2, and Δ on the performance of the device.

This generator is a variant of the "separately excited" MHD converter, whose analysis is presented elsewhere,[2,3] which, in fact, also displays a self-excited mode of operation. In connection with that analysis, we observed that the equation previously stated can be treated analytically to the point of obtaining algebraic expressions that define the trajectories of the solutions in the phase space (x, $\dot{x}$, $\ddot{x}$) of the differential equations. In the terminology employed in the note, these are

$$(C_2 - x)\ddot{x} - \tfrac{1}{2}\dot{x}^2 + 2C_1\dot{x} = D_1 = \Delta C_2 - \tfrac{1}{2} + 2C_1$$
$$\ddot{x} = D_2(\dot{x} - a)^{\alpha}(b - \dot{x})^{\beta}$$

where D_1, D_2, a, b, α, and β are constants which depend only on the system parameters C_1, C_2, and Δ. These relations form a sufficient basis for a quantitative analytical study and a consequent evaluation of the performance of the device, without recourse to numerical integration. This study will appear in a forthcoming publication.[4]

Some points in the note do, however, merit immediate comment. The authors' expression for the conversion efficiency $2|\Delta|C_2/(C_2 - 1)$ will yield values in excess of unity unless the magnitude of Δ is required not to exceed the limit

$$\Delta_m = (C_2 - 1)/2C_2$$

To resolve this difficulty, it should be pointed out that, with sufficiently large electrical excitation ($|\Delta|$) and magnetic Reynolds number ($1/C_1$), the current and force build up rapidly, causing the slug to be stopped before reaching the end of the channel, and to be propelled back towards the entry to the channel. Under these conditions, the derivation of the expression for the efficiency is not valid: the final velocity must be evaluated at $x = 0$ and not at $x = 1$. With an infinite magnetic Reynolds number, this condition arises when $|\Delta| > \Delta_m$. However, unit efficiency is obtained with $|\Delta| = \Delta_m$, and a further reduction of the excitation must be accompanied by a sacrifice of efficiency. Thus, in order to approach the absolute limit of self-excitation $\Delta = 0$ without sacrificing efficiency, one must strive to make C_2 as close to 1 as possible. Contrary to the authors' contention, setting $C_2 = 1$ does not give rise to any mathematicall discontinuity at the downstream end of the channel, nor does it pose any difficulty in the treatment of the equations. The correct statement is that the limiting behavior of the system as C_2 approaches arbitrarily close to 1 is not the behavior obtained by solving the equations with $C_2 = 1$ (i.e., we have a singular perturbation).

Moreover, although the note concentrates on the electromechanical conversion efficiency, inadequate attention is paid to the division of the converted energy between the load and the slug. The note is, in fact, very unclear as to the meaning of the symbol R, which is labelled as resistance but is defined as the slug resistance only in the course of the discussion (the sketch does show a load resistance R_l, but no mention of it is made in the text). The fact is that the ratio R_l/R_{slug} plays a significant role in the optimization considerations. In order to provide the best energy transfer to the load, this ratio should not be set to unity, as in an ordinary linear circuit, except in the limit $C_1 \to \infty$, i.e., when the magnetic Reynolds number approaches 0. This is so, because the device is nonlinear and cannot be represented by a Thevenin equivalent with a fixed internal resistance equal to that of the slug.

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* Project Scientist, Geophysics Division.

References

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[2] Frankenthal, S., "The performance of thermochemically driven MHD converters," Pt. I (to be published).
[3] Treve, Y. M., "The performance of thermochemically driven MHD converters," Pt. II (to be published).
[4] Frankenthal, S., "The series-excited MHD converter," AIAA J. (submitted for publication).

Reply by Authors to S. Frankenthal

A. L. LOEFFLER JR.,* L. ARONOWITZ,† AND W. B. ERICSON*
Grumman Aircraft Engineering Corporation, Bethpage, N. Y.

FRANKENTHAL'S comments seem not to be in any substantial disagreement with our note, but rather are mainly concerned with topics that we did not incorporate and are considered in some of his soon-to-be-published works. We appreciate his comments and look forward to the full publication of his analysis.

His comment concerning the possibility that under certain circumstances the plasma slug will stop and reverse direction before reaching the end of the generator should perhaps be quite evident from Eq. (5b) of our note,

$$v^2 = 1 + 2\Delta C_2 x/(C_2 - x)$$

and the $\Delta = -0.1$ curves of our Fig. 2, which demonstrate such conditions. Also our expression for the efficiency of energy transfer,

$$1 - v^2 = -2\Delta C_2/(C_2 - 1)$$

explicitly assumes that the plasma slug travels the total length of the generator. Frankenthal's comments emphasizing the condition for total transfer of kinetic energy to electrical energy, namely that obtained by setting $v^2 = 0$ in the preceding equation, are certainly important. A major problem from a practical point of view, however, will be that of ob-

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* Co-Head of MHD and Plasma Physics Group, Research Department.
† Research Scientist, Research Department. Member AIAA.

taining a large enough Δ (numerically equal to one half the ratio of initial electrical to kinetic energy) and small enough C_2 (ratio of total initial inductance to generator inductance) for a reasonably operative system.

We apologize for not explicitly spelling out in our note that the resistance R always denotes the total resistance, $R_{\text{slug}} + R_l$. As we showed, however, it is only for the case of $R_l = 0$ that C_1 becomes identical to the familiar magnetic Reynolds number. For $R_l \neq 0$, it can be considered a "generalized" magnetic Reynolds number. The question of optimizing generator action by suitably matching R_l and R_{slug} is an interesting one. We did not attempt to answer it in our brief note beyond stating that some sort of impedance matching network will generally be required to maximize the transfer of energy to the load.

It should be emphasized that one of our major findings was that for successful operation of the system the total resistance should be quite small. In fact, the only feasible mode of operation of such a system may be one where the load resistance is short circuited during the deceleration of the plasma slug and switched in after the slug has given up its kinetic energy.

We stand corrected on the inappropriate use of the word "discontinuity" for describing the singularity associated with $C_2 = 1$.

Some Heat Conduction Analyses Using Kantorovich's Method

D. J. Johns*
Loughborough University of Technology, Leicestershire England

THE temperature distribution in an isotropic solid in which the specific heat C and the thermal conductivity k can vary with temperature may be determined from the partial differential equation[1]

$$(C/k)(\partial\theta/\partial t) = \nabla^2\theta \tag{1}$$

This equation refers specifically to the case of a solid of thermal conductivity equal to unity and a heat capacity C/k, a function of θ, the temperature.

Kantorovich's method[2] is based on Eq. (1), and with the assumption that θ can be written in terms of unknown parameters q_i, the following simultaneous equations must be solved for one-dimensional heat flow problems:

$$\int[(\partial^2\theta/\partial y^2) - (C/k)(\partial\theta/\partial t)](\partial\theta/\partial q_i)\cdot dy = 0 \tag{2}$$

To illustrate the application of Kantorovich's method, consider a slab of thickness l which has one face at $y = 0$, suddenly heated to a temperature θ_0 which then is maintained. The other face at $y = l$ is thermally insulated. This problem has already been considered in analyses by Biot,[3] with which comparisons now are made. A convenient expression for the temperature distribution during the initial phase of heating of the slab is:

$$\theta = \theta_0[1 - y/q_1]^2 \qquad 0 < y < q_1 \tag{3}$$

where $q_1(t)$ is the generalized co-ordinate known as the "penetration depth," which is a function of time t.

For constant values of k and C, substitution of (3) into (2) yields, eventually,

$$q_1^2 = 10Kt \tag{4}$$

where K = thermal diffusivity = k/C. The corresponding coefficient from Ref. 3 is shown to be 11.3. The first phase of heating ends when $q_1 = l$ and the temperature starts to rise at the insulated surface $y = l$. For this second phase of heating, the temperature distribution may be approximated by:

$$\theta = \theta_2 + (\theta_0 - \theta_2)[1 - y/l]^2 \tag{5}$$

where θ_2 (t) = the temperature rise at the insulated surface and is the appropriate generalised co-ordinate q_2 in Eq. (2). Substitution of Eq. (5) into Eq. (2) leads to the following differential equation in θ_2:

$$\theta_2 + 0.4(l^2/K)\dot{\theta}_2 = \dot{\theta}_0 \tag{6}$$

which agrees almost exactly with Ref. 3.

For the case of a slab with temperature-dependent heat capacity we will assume that

$$C = C_0[1 + (\theta/\theta_0)^m] \qquad 0 \leqslant m \leqslant \infty \tag{7}$$

i.e., C varies by a factor of 2 within the range of temperatures considered. We define $K_0 = k/C_0$. Substitution of (7) and (3) into (2) yields

$$q_1 = A(K_0 t)^{1/2} \tag{8}$$

where

$$A = 3.16[1 + 30/(2m + 3)(2m + 5)(m + 2)]^{-1/2} \tag{9}$$

and the specific results shown in Table 1 are obtained.

Table 1 Results for temperature dependent heat capacity from Eqs. (7) and (8)

m	C/C_0	A [Eq. (9)]	A (Ref. 3)
0	2	2.23	2.37
∞	1	3.16	3.36
1	$1 + \theta/\theta_0$	2.79	2.97

Alternatively, if Eq. (9) is replaced by

$$C = C_0[1 + r(\theta/\theta_0)] \tag{10}$$

i.e., C might vary by more than a factor of 2, the corresponding end result is

$$q_1 = 3.16(K_0 t)^{1/2}[1 + (r/3.5)]^{-1/2} \tag{11}$$

It can be shown that the heat penetration depth q_1 can be obtained from the case with constant properties ($r = 0$) by using the value $C = C_0$ $[1 + (r/3.5)]$, e.g., for $r = 1$, $C = 1.28$ C_0. This may be appreciably lower than the average value $C = C_0[1 + (r/2)]$, and indicates that the heat propagation is controlled more by the value of C in the region of lower temperature. This result also was shown in Ref. 3.

For the case of a slab with temperature-dependent thermal conductivity we will assume that

$$k = k_0[1 + S(\theta/\theta_0)] \tag{12}$$

Using the temperature distribution of Eq. (3), we obtain the final result

$$q_1 = B(k_0 t/C)^{1/2} \tag{13}$$

where

$$B = [10(1 + 0.9S)]^{1/2} \tag{14}$$

Table 2 Results for temperature dependent thermal conductivity from Eqs. (12) and (13)

S	B [Eq. (18)]	B (Ref. 4)
$\frac{1}{2}$	3.81	3.83
0	3.16	3.36
$-\frac{1}{2}$	2.34	2.76

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* Reader in Aeronautical Engineering. Associate Fellow AIAA.